Keynesian model with public and external sectors

We consider the Keynesian model with public and external sectors:

$$C = 1000 + 0.8\,Y_d, \quad I = 500, \quad G = 200, \quad T = 100 + 0.1\,Y, \quad \text{Tr} = 200$$

1. Find the equilibrium income Y in the closed economy model (without external sector).
2. Calculate the fiscal surplus, that is, $T - (G + \text{Tr})$.
3. $$X = 800, \quad M = 500 + 0.1\,Y$$

   Find the equilibrium income Y in the open economy model, including X and M.
4. Determine the trade balance, $X - M$.
5. Assuming an increase in public spending of 1000 (i.e., $G' = G + 1000$), find the new income Y', the new fiscal surplus, and the new trade balance.

Solution

1. **Equilibrium income (closed economy).** At equilibrium $Y = C + I + G$ and $Y_d = Y - T + \text{Tr}$.

$$Y = 1000 + 0.8\,(Y - (100 + 0.1Y) + 200) + 500 + 200$$

Calculating:

$$Y = 1000 + 0.8\,(0.9Y + 100) + 700 = 1000 + 0.72Y + 80 + 700 = 1780 + 0.72Y$$

$$Y - 0.72Y = 1780 \quad \Longrightarrow \quad 0.28Y = 1780 \quad \Longrightarrow \quad Y = 6357.14$$

2. **Fiscal surplus.** The revenue is $T = 100 + 0.1Y$, and the total spending is $G + \text{Tr}$.

$$T = 100 + 0.1 \cdot 6357.14 = 735.71, \qquad G + \text{Tr} = 200 + 200 = 400$$

$$\text{Fiscal surplus} = T - (G + \text{Tr}) = 335.71$$

3. **Equilibrium income (open economy).** Now $Y = C + I + G + (X - M)$. With $Y_d = 0.9Y + 100$,

$$C = 1000 + 0.8\,(0.9Y + 100) = 1080 + 0.72Y, \quad X - M = 800 - (500 + 0.1Y) = 300 - 0.1Y$$

Then

$$Y = (1080 + 0.72Y) + 500 + 200 + (300 - 0.1Y) = 2080 + 0.62Y$$

$$Y - 0.62Y = 2080 \quad \Longrightarrow \quad 0.38Y = 2080 \quad \Longrightarrow \quad Y = 5473.68$$

4. **Trade balance.**

$$X - M = 300 - 0.1 \cdot 5473.68 = 300 - 547.37 = -247.37$$

5. **Effect of $\Delta G = +1000$.** Let $G' = 200 + 1000 = 1200$. Repeating (c):

$$Y' = (1080 + 0.72Y') + 500 + 1200 + (300 - 0.1Y') = 3080 + 0.62Y' \quad \Longrightarrow \quad 0.38Y' = 3080 \quad \Longrightarrow \quad Y' = 8105.26$$

Revenue:

$$T' = 100 + 0.1 \cdot 8105.26 = 910.53, \quad G' + \text{Tr} = 1200 + 200 = 1400$$

$$\text{Fiscal surplus}' = 910.53 - 1400 = -489.47$$

Trade balance:

$$X - M' = 800 - (500 + 0.1 \cdot 8105.26) = 800 - 1310.53 = -510.53$$